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Casimir Effect Under Two Dimensional Black Hole Spacetime Background with Global Monopole

Xiao-Xia Wang[1] and Shu-Zheng Yang[1]

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A thin layer of the event horizon vicinity to the two-dimension black hole with a global monopole is considered as a system of the Casimir type. The energy-momentum tensor is derived in Boulware vacuum, Hartle-Hawking vacuum and Unruh vacuum respectively. The values are derived in the massless scalar field which satisfies the Dirichlet boundary conditions. Using the Wald's axioms, the result is got which is the same with the one derived by the usual regularized methods. Meanwhile, the energy, energy density, and pressure acting on the boundaries at the asymptotically flat background also are calculated too, and from the energy, Casimir force is derived. The Casimir energy and Casimir force are compared respectively in the background before and after radiation.

KEY WORDS: casimir effect; two dimensional black hole; global monopole.
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1. INTRODUCTION

Using the physical picture of the vacuum fluctuation, in 1948, Casimir first predicted that two uncharged conducting parallel plates would experience an attractive force, afterward, the force is called Casimir force. This phenomenon is called the Casimir effect (Casimir, 1948) which has abroad applications in atomic physics, condensed matter physics, particles and gravitation physics and so on. The importance of Casimir effect is being understood gradually. The Casimir effect has been confirming by the gradually precise experimentation, as an observable effect. Heretofore a great deal of work on Casimir effect has been done both experimentally and theoretically.

There are many factors affecting the Casimir effect, such as constrains,the space topology, temperature etc., and the flat background or the curved background is one of the important ones. In reference (Christodoulakis, Diamandis, and Getorgalas, 2001; Elias and Vagenas, 2003; Gao and Zhu, 2005) the Casimir effect of the massless scalar field has been investigated in several different backgrounds.

[1] Institute of Theoretical Physics, China West Normal University, Nanchong, Sichuan 637002, China; e-mail: xiaoxiawang2004@163.com.

Although lots of work about the black hole tunneling has been done recently (Yang, Jiang, and Li, 2005; Jiang, Yang, and Li, 2006; Chen, Jiang, Li, and Yang, 2006), no one has investigated the Casimir effect in the black hole background after tunneling. In this paper, we will investigate the Casimir effect in the black hole background after tunneling with a global monopole.

In this paper, considering a thin layer of the event horizon vicinity to the given black hole background as a system of the Casimir style, we investigate the Casimir effect of the massless scalar field in the global monopole background after tunneling. When we investigate the Casimir effect, we must calculate the renormalized energy-moment tensor. Although there are many regularized methods, such as Green's function method, zeta function regularization, dimensional regularization, and point-splitting method etc, it is very difficult to calculate the energy-moment tensor of the constrained field. We only do our research in the two-dimension spacetime. This two-dimension black hole is asymptotically flat, satisfying Dirichlet boundary conditions. We do our research applying the Wald's axioms (Wald, 1977; Wald, 1978) instead of the usual regularized methods.

This paper is organized as follows, in section II the geometrical property of the given black hole will be described and the useful geometrical quantities will be calculated too. And the general form of the energy momentum tensor is derived. In section III, The expectation values of the renormalized energy momentum tensors for the massless scalar field in the given background will be calculated respectively in three different vacua, which are Boulware vacuum, Hartle-Hawking vacuum and Unruh vacuum. Meanwhile, we will calculate the energy, energy density, and pressure acting on the boundaries at the asymptotically flat background, and from the energy, Casimir force is derived too. Finally, the results will be discussed.

2. THE SPACETIME BACKGROUND AND THE GENERAL FORM OF THE ENERGY MOMENTUM TENSOR

After tunneling particles with the mass$\omega$and the charge$q$, the line element of the black hole with a global monopole is

$$ds^2 = -\left[1 - 8\pi\eta^2 - \frac{2(m-\omega)}{r} + \frac{(Q-q)^2}{r^2}\right]dt^2 + \left[1 - 8\pi\eta^2 - \frac{2(m-\omega)}{r} + \frac{(Q-q)^2}{r^2}\right]^{-1} dr^2, \tag{1}$$

where m and η are the ADM mass and the scale of the symmetry breaking of the black hole. From the equation of the null superface, we can get the radius of the even horizon as

$$r_H = \frac{m - \omega + \sqrt{(m-\omega)^2 - (1-8\pi\eta^2)(Q-q)^2}}{1 - 8\pi\eta^2}. \quad (2)$$

According to the surface gravitation κ of this black hole, the Hawking temperature T_H of the black hole horizon is derived as

$$T_H = \frac{\kappa}{2\pi k_B} = \frac{(1-8\pi\eta^2)^2\sqrt{P}}{2\pi}, \quad (3)$$

where

$$P = \frac{(m-\omega)^2 - (1-8\pi\eta^2)(Q-q)^2}{\left[m - \omega + \sqrt{(m-\omega)^2 - (1-8\pi\eta^2)(Q-q)^2}\right]^4}.$$

Since the conformal transformation do not change the causality of the spacetime, in order to get the best conclusions in the Minkowski spacetime, we make conformal transformation to the Eq. (1), and get

$$ds^2 = f(r)(-dt^2 + dR^2), \quad (4)$$

where

$$f(r) = 1 - 8\pi\eta^2 - \frac{2(m-\omega)}{r} + \frac{(Q-q)^2}{r^2} = \frac{dr}{dR}. \quad (5)$$

From the Eq. (4), we get the non-zero Christoffel symbols of the line element as

$$\Gamma^R_{tt} = \Gamma^t_{tR} = \Gamma^t_{Rt} = \Gamma^R_{RR} = \frac{1}{2}\frac{\partial f(r)}{\partial r} = \frac{(m-\omega)r - (Q-q)^2}{r^3}. \quad (6)$$

Further, the Ricci scalar of the line element (4) is given as

$$R(r) = \frac{-2[2(m-\omega)r - 3(Q-q)^2]}{r^4}. \quad (7)$$

In the process of regularization, the trace is non-zero. It is given in the two dimensions as follows (Capri, Kobayashi, and Lamb, 1996).

$$T^\alpha_\alpha(r) = \frac{R(r)}{24\pi}. \quad (8)$$

From the Eq. (7) and the Eq. (8), we get the trace of the energy-momentum tensor for the two-dimension given black hole as

$$T^\alpha_\alpha(r) = \frac{-[2(m-\omega)r - 3(Q-q)^2]}{12\pi r^4}. \quad (9)$$

Applying the Wald's axioms, the renormalized energy-momentum tensor satisfies the conservation equation

$$T^\mu_{\nu;\mu} = 0, \quad (10)$$

which contains two equations (Christodoulakis, Diamandis, and Getorgalas, 2001; Elias and Vagenas, 2003; Gao and Zhu, 2005).

$$\frac{dT_t^R}{dR} + \Gamma_{RR}^R T_t^R - \Gamma_{tt}^R T_R^t = 0, \tag{11}$$

$$\frac{dT_R^R}{dR} + \Gamma_{tR}^t T_R^R - \Gamma_{Rt}^t T_t^t = 0, \tag{12}$$

Since $T_R^t = -T_t^R$ and $T_t^t = T_\alpha^\alpha - T_R^R$, we get

$$\frac{dT_t^R}{dR} + 2\Gamma_{tR}^t T_t^R = 0, \tag{13}$$

$$\frac{dT_R^R}{dR} + 2\Gamma_{tR}^t T_R^R = \Gamma_{tR}^t T_R^R. \tag{14}$$

Substituting Eq. (6) into Eq. (13), we can get

$$\frac{d}{dr}\left(f\left(r\right)T_t^R\right) = 0. \tag{15}$$

The solution of Eq. (15) is

$$T_t^R = \alpha f^{-1}\left(r\right), \tag{16}$$

where α is a constant of integration. In the same way Eq. (14) becomes

$$\frac{d}{dr}\left(f(r)T_R^R\right) = \frac{1}{2}\frac{\partial f(r)}{\partial r}T_\alpha^\alpha. \tag{17}$$

The solution of Eq. (17) is

$$T_R^R = \frac{1}{f\left(r\right)}\left[H\left(r\right) + \beta\right], \tag{18}$$

where

$$H(r) = \frac{1}{2}\int_{r_H}^{r}\frac{df(r')}{dr'}T_\alpha^\alpha(r')dr'. \tag{19}$$

Likewise, β is a constant of integration, while the point r_H is where the event horizon is placed. Equations (2), (6) and (9) being used, Eq. (19) becomes

$$H\left(r\right) = \frac{1}{24\pi}\left[\frac{(m-\omega)^2}{r^4} - \frac{2\left(m-\omega\right)\left(Q-q\right)^2}{r^5} + \frac{\left(Q-q\right)^4}{r^6}\right] - D, \tag{20}$$

where

$$D = \frac{1}{24\pi}\left[\frac{(m-\omega)^2}{r_+^4} - \frac{2\left(m-\omega\right)\left(Q-q\right)^2}{r_+^5} + \frac{\left(Q-q\right)^4}{r_+^6}\right]$$

and $r_+$is the outer event horizon of the given black hole. The limiting values of $H(r)$ from Eq. (20) are given as follows

$$\text{if} \quad r \to -\infty \, (r \to r_H) \, , \quad \text{then} \quad H\,(r) = 0.$$

$$\text{if} \quad r \to +\infty, \quad \text{then}$$

$$H(r) = -\frac{1}{24\pi}\left[\frac{(m-\omega)^2}{r_+^4} - \frac{2\,(m-\omega)\,(Q-q)^2}{r_+^5} + \frac{(Q-q)^4}{r_+^6}\right] = -D.$$

In any two-dimension background, the general form of the energy-momentum tensor can be expressed

$$T_\upsilon^\mu = \begin{bmatrix} T_\alpha^\alpha\,(r) - f^{-1}\,(r)\,H\,(r) & 0 \\ 0 & f^{-1}\,(r)\,H\,(r) \end{bmatrix} + f^{-1}\,(r)\begin{bmatrix} -\beta & -\alpha \\ \alpha & \beta \end{bmatrix}. \quad (21)$$

In the Eq. (21) the only unknown parameter is α and β. We will apply the Wald's axioms to determine their values.

We restrict the massless scalar field between r_H and $r'_H(r'_H = r_H + L)$. This massless scalar field satisfies Dirichlet boundary conditions at r_H and r'_H.

Next we will investigate the renormalized energy-momentum tensor in different vacua.

3. THE ENERGY-MOMENTUM TENSOR IN DIFFERENT VACUA

3.1. Boulware Vacuum

In this vacuum (Hang, Zhou, and Zhang, 1994), there is no particles at infinity. The renormalized energy-momentum tensor should coincide at infinity with the standard energy-momentum tensor in the Minkowski spacetime

$$T_\nu^\mu = \frac{\pi}{24L^2}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (22)$$

Since the given black hole is asymptotically flat at infinity, i.e. at infinity is Minkowski spacetime. When $r \to +\infty$, the Eq. (21) should coincide with the Eq. (22). Thus we can get the values of the parameter α and β

$$\alpha = 0 \quad \beta = \frac{\pi(1 - 8\pi\eta^2)}{24L^2} + D. \quad (23)$$

So in the Boulware vacuum, the renormalized energy-momentum tensor is

$$T_\nu^{(\eta)\mu} = \begin{bmatrix} T_\alpha^\alpha\,(r) - f^{-1}\,(r)\,H\,(r) & 0 \\ 0 & f^{-1}\,(r)\,H\,(r) \end{bmatrix}$$

$$+f^{-1}(r)\left[\frac{\pi(1-8\pi\eta^2)}{24L^2}+D\right]\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}. \tag{24}$$

The Eq. (24) can be written as

$$T_{\upsilon}^{(\eta)\mu}=T_{\nu(\text{gravitational})}^{\mu}+T_{\nu(\text{boundary})}^{\mu} \tag{25}$$

where η denotes that the renormalized energy-momentum tensor is calculated in the Boulware vacuum. The first term of the Eq. (25) denotes the contribution due to the gravitation background and the second term denotes the contribution due to the presence of the boundary.

When $r\to+\infty$, the spacetime is asymptotically flat. In Boulware vacuum, the energy density, pressure, and energy between the boundaries are given by

$$\rho=T_t^{(\eta)t}=-\frac{\pi}{24L^2}, \tag{26}$$

$$p=-T_R^{(\eta)R}=-\frac{\pi}{24L^2}, \tag{27}$$

$$E(L)=\int_{r_1}^{r_1+L}\rho dR=-\frac{\pi}{24L}. \tag{28}$$

The corresponding Casimir force F between the boundaries is

$$F(L)=-\frac{\partial E(L)}{\partial L}=-\frac{\pi}{24L^2}<0 \tag{29}$$

which is attractive force.

3.2. Hartle-Hawking Vacuum

Hartle-Hawking vacuum (Hang, Zhou, and Zhang, 1994) is the conformal vacuum relevant to the homogeneous gravitation field. In Hartle-Hawking vacuum, the black hole is in thermal equilibrium state with the temperature T. The standard energy-momentum tensor (22) will be modified by

$$T_{\nu}^{\mu}=\frac{\pi T^2}{12}\begin{bmatrix}-2 & 0\\ 0 & 2\end{bmatrix}=\frac{\pi T^2}{6}\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}, \tag{30}$$

in which T is the Hawking temperature T_H of this two-dimensional given black hole. The standard energy-momentum tensor becomes

$$T_{\nu}^{\mu}=\frac{\pi}{24L^2}\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}+\frac{(1-8\pi\eta^2)^4P}{24\pi}\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}. \tag{31}$$

When $r \to +\infty$, the Eq. (21) should coincide with the Eq. (31). Thus we can get the values of the parameter α and β

$$\alpha = 0, \quad \beta = \frac{\pi(1-8\pi\eta^2)}{24L^2} + \frac{(1-8\pi\eta^2)^5 P}{24\pi} + D. \tag{32}$$

The renormalized energy-momentum tensor is

$$T_\nu^{(\upsilon)\mu} = \begin{bmatrix} T_\alpha^\alpha(r) - f^{-1}(r)H(r) & 0 \\ 0 & f^{-1}(r)H(r) \end{bmatrix} + f^{-1}(r)\left[\frac{\pi(1-8\pi\eta^2)}{24L^2} + \frac{(1-8\pi\eta^2)^5 P}{24\pi} + D\right]\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{33}$$

where υ denotes that the renormalized energy-momentum tensor is calculated in Hartle-Hawking vacuum. Thus the Eq. (33) can be written as

$$T_\nu^{(\upsilon)\mu} = T^\mu_{\nu(\text{gravitational})} + T^\mu_{\nu(\text{boundary})} + T^\mu_{\nu(bath)}, \tag{34}$$

where the last term denotes the contribution due to the thermal bath at temperature T_H.

When $r \to +\infty$, in Hartle-Hawking vacuum the energy density, pressure, and energy between the boundaries are given as

$$\rho = T_t^{(\upsilon)t} = -\left[\frac{\pi}{24L^2} + \frac{(1-8\pi\eta^2)^4 P}{24\pi}\right], \tag{35}$$

$$p = -T_R^{(\upsilon)R} = -\left[\frac{\pi}{24L^2} + \frac{(1-8\pi\eta^2)^4}{24\pi}P\right], \tag{36}$$

$$E(L,T_H) = \int_{r_1}^{r_1+L} \rho dR = -\left[\frac{\pi}{24L} + \frac{(1-8\pi\eta^2)^4 LP}{24\pi}\right] = -\left(\frac{\pi}{24L} + \frac{\pi L}{6}T_H^2\right). \tag{37}$$

The corresponding Casimir force F between the boundaries is

$$F(L,T_H) = -\left(\frac{\partial E(L,T_H)}{\partial L}\right)_{T_H} = -\frac{\pi}{24L^2} + \frac{(1-8\pi\eta^2)^4 P}{24\pi} = -\frac{\pi}{24L^2} + \frac{\pi}{6}T_H^2, \tag{38}$$

which is not always attractive force. It is clear that the Casimir force is

(1) attractive

$$\text{when } L < \frac{1}{2T_H} = \frac{\pi}{(1-8\pi\eta^2)^2\sqrt{P}}, \tag{39}$$

(2) zero

$$\text{when } L = \frac{1}{2T_H} = \frac{\pi}{(1 - 8\pi\eta^2)^2\sqrt{P}}, \tag{40}$$

(3) repulsive

$$\text{when } L > \frac{1}{2T_H} = \frac{\pi}{(1 - 8\pi\eta^2)^2\sqrt{P}}, \tag{41}$$

which is decided by the distance L, and related to the scale of the symmetry breaking η.

3.3. Unruh vacuum

Unruh vacuum (Hang, Zhou, and Zhang, 1994) is considered as a vacuum, in which the two-dimensional given black hole is in a thermal state at Hawking temperature T_H. Because of Hawking radiation, an outward flux is detected at infinity in this vacuum state. The standard energy-momentum tensor (22) will be modified by an additional term

$$T_\nu^\mu = \frac{\pi T_H^2}{12}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} = \frac{(1 - 8\pi\eta^2)^4 P}{48\pi}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}. \tag{42}$$

The renormalized energy-momentum tensor (21) should now coincide at infinity with the following stress tensor

$$T_\nu^\mu = \frac{\pi}{24L^2}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{(1 - 8\pi\eta^2)^4 P}{48\pi}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}. \tag{43}$$

Therefore we get

$$\alpha = \frac{(1 - 8\pi\eta^2)^4 P}{48\pi}, \tag{44}$$

$$\beta = \frac{\pi(1 - 8\pi\eta^2)}{24L^2} + \frac{(1 - 8\pi\eta^2)^5 P}{48\pi} + D. \tag{45}$$

Thus the renormalized energy-momentum tensor $T_\nu^{(\xi)\mu}$ becomes

$$T_\nu^{(\xi)\mu} = \begin{bmatrix} T_\alpha^\alpha(r) - f^{-1}(r) H(r) & 0 \\ 0 & f^{-1}(r) H(r) \end{bmatrix} + f^{-1}(r)$$

$$\times \begin{bmatrix} -\frac{\pi\left(1 - 8\pi\eta^2\right)}{24L^2} - \frac{\left(1 - 8\pi\eta^2\right)^5 P}{48\pi} - D & -\frac{\left(1 - 8\pi\eta^2\right)^4 P}{48\pi} \\ \frac{\left(1 - 8\pi\eta^2\right)^4 P}{48\pi} & \frac{\pi\left(1 - 8\pi\eta^2\right)}{24L^2} + \frac{\left(1 - 8\pi\eta^2\right)^5 P}{48\pi} + D \end{bmatrix}.$$

(46)

where ξ denotes that the renormalized energy-momentum tensor is calculated in Unruh vacuum. The Eq. (46) can be written as

$$T_{\nu}^{(\xi)\mu} = T^{\mu}_{\nu(\text{gravitational})} + T^{\mu}_{\nu(\text{boundary})} + T^{\mu}_{\nu(\text{radiation})}, \tag{47}$$

where the last term represents the contribution due to Hawking radiation at the temperature T_H.

In this vacuum, when $r \to \infty$, the detected energy density, pressure, and energy between the boundaries are given as

$$\rho = T_t^{(\xi)t} = -\left[\frac{\pi}{24L^2} + \frac{(1-8\pi\eta^2)^4 P}{48\pi}\right], \tag{48}$$

$$p = -T_R^{(\xi)R} = -\left[\frac{\pi}{24L^2} + \frac{(1-8\pi\eta^2)^4}{48\pi} P\right], \tag{49}$$

$$\begin{aligned} E(L, T_H) &= \int_{r_1}^{r_1+L} \rho dR = -\left[\frac{\pi}{24L} + \frac{(1-8\pi\eta^2)^4 LP}{48\pi}\right] \\ &= -\left(\frac{\pi}{24L} + \frac{\pi L}{12} T_H^2\right). \end{aligned} \tag{50}$$

The Casimir force F between the boundaries is

$$\begin{aligned} F(L, T_H) &= -\left(\frac{\partial E(L, T_H)}{\partial L}\right)_{T_H} = -\frac{\pi}{24L^2} + \frac{(1-8\pi\eta^2)^4 P}{48\pi} \\ &= -\frac{\pi}{24L^2} + \frac{\pi}{12} T_H^2. \end{aligned} \tag{51}$$

It is not always attractive too. It can be

(1) attractive

$$\text{when } L < \frac{1}{\sqrt{2}T_H} = \frac{\sqrt{2}\pi}{(1-8\pi\eta^2)^2\sqrt{P}}, \tag{52}$$

(2) zero

$$\text{when } L = \frac{1}{\sqrt{2}T_H} = \frac{\sqrt{2}\pi}{(1-8\pi\eta^2)^2\sqrt{P}}, \tag{53}$$

(3) repulsive

$$\text{when } L > \frac{1}{\sqrt{2}T_H} = \frac{\sqrt{2}\pi}{(1-8\pi\eta^2)^2\sqrt{P}}. \tag{54}$$

which is decided by the distance L, and related to the scale of the symmetry breaking η.

Removing the last term in Eqs. (38) and (51) respectively, the net force can be derived. The reason is that in both vacua (Hartle-Hawking and Unruuh vacua) the forces contributed by the thermal bath or radiation acting on both sides of each boundary are the same, and their total contribution to the net force is nothing. So the net force acting on the Dirichlet walls is

$$F_{\mathrm{net}} = -\frac{\pi}{24L^2} \tag{55}$$

It is obvious that the net force always attractive.

4. DISCUSSION AND CONCLUSIONS

Before the black hole tunneling particles, the two dimensional line element is

$$ds^2 = -\left(1 - 8\pi\eta^2 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - 8\pi\eta^2 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2. \tag{56}$$

For Boulware vacuum in the background before tunneling, the Casimir energy is

$$E'_\eta(L) = \int_{r_1}^{r_1+L} \rho dR = -\frac{\pi}{24L}, \tag{57}$$

and the Casimir force is

$$F'_\eta(L) = -\frac{\partial E(L)}{\partial L} = -\frac{\pi}{24L^2} < 0. \tag{58}$$

In this vacuum, the Casimir force and Casimir energy are both the same in the black hole background before and after tunneling.

For Hartle-Hawking vacuum, the Casimir energy is

$$\begin{aligned} E'_\nu(L, T_H) &= \int_{r_1}^{r_1+L} \rho dR = -\left[\frac{\pi}{24L} + \frac{(1-8\pi\eta^2)^4 LP}{24\pi}\right] \\ &= -\left(\frac{\pi}{24L} + \frac{\pi L}{6}T'^2_H\right) \end{aligned} \tag{59}$$

and the Casimir force is

$$\begin{aligned} F'_\nu(L, T_H) &= -\left(\frac{\partial E(L, T_H)}{\partial L}\right)_{T_H} = -\frac{\pi}{24L^2} + \frac{(1-8\pi\eta^2)^4 P}{24\pi} \\ &= -\frac{\pi}{24L^2} + \frac{\pi}{6}T'^2_H \end{aligned} \tag{60}$$

where

$$T'_H = \frac{\kappa}{2\pi k_B} = \frac{\left(1 - 8\pi\eta^2\right)^2 \sqrt{P'}}{2\pi},$$

and

$$P' = \frac{m^2 - (1 - 8\pi\eta^2)Q^2}{[m + \sqrt{m^2 - (1 - 8\pi\eta^2)Q^2}]^4}.$$

In this vacuum, the difference of the Casimir force in the black hole background before and after tunneling is

$$\Delta E_\nu = -\frac{\pi L}{6}\left(T_H^2 - T'^2_H\right), \tag{61}$$

and the difference of the Casimir energy in the black hole background before and after tunneling is

$$\Delta F_\nu = \frac{\pi L}{6}\left(T_H^2 - T'^2_H\right) \tag{62}$$

Comparing the Eqs. (59) and (60) with the Eqs. (37) and (38), we can see that the Casimir force and Casimir energy get larger in the black hole background after tunneling. This phenomen is caused by the self-gravitation which leads the shrinking of the black hole event horizon. In Unruh vacuum, the Casimir energy is

$$\begin{aligned} E'_\xi(L, T_H) &= \int_{r_1}^{r_1+L} \rho dR = -\left[\frac{\pi}{24L} + \frac{(1 - 8\pi\eta^2)^4 L P}{48\pi}\right] \\ &= -\left(\frac{\pi}{24L} + \frac{\pi L}{12} T'^2_H\right) \end{aligned} \tag{63}$$

and the Casimir force is

$$\begin{aligned} F'_\xi(L, T_H) &= -\left(\frac{\partial E(L, T_H)}{\partial L}\right)_{T_H} = -\frac{\pi}{24L^2} + \frac{(1 - 8\pi\eta^2)^4 P}{48\pi} \\ &= -\frac{\pi}{24L^2} + \frac{\pi}{12} T'^2_H \end{aligned} \tag{64}$$

In this vacuum, the difference of the Casimir force in the black hole background before and after tunneling is

$$\Delta E_\xi = -\frac{\pi L}{12}\left(T_H^2 - T'^2_H\right) \tag{65}$$

and the difference of the Casimir energy in the black hole background before and after tunneling is

$$\Delta F_\xi = \frac{\pi L}{12}\left(T_H^2 - T'^2_H\right) \tag{66}$$

Comparing the Eqs. (63) and (64) with the Eqs. (50) and (51), we can see that the Casimir force and Casimir energy are getting larger too in the black hole background after tunneling, because of the self-gravitation.

From Ref. (Setare, 2001) we can know that if the energy-momentum tensor of a certain field with one exterior boundary in the Minkowski spacetime can be obtained, it can also be obtained for the same field with the same boundary in curved spacetime.

In this paper, we explicitly calculated the renormalized energy-momentum tensor of a massless scalar field which satisfies Dirichlet boundary conditions in the black hole spacetime backgroud after tunneling with a global monopole. The renormalized energy momentum tensor is treated in the Boulware, Hartle-Hawking, and Unruh vacua separately. In all these vacua, when $r \to +\infty$, the energy density, pressure and energy acting on boundaries are obtained. The values of the above-mentioned quantities are all negative. We also calculate the Casimir force, which is attractive in Boulware vacuum. But in Hartle-Hawking and Unruh vacua it can be attractive, repulsive, and zero according to the distance L between the boundaries, and is related to the scale of the symmetry breaking η.

We evaluated the net force exerted on the boundaries too, which is always negative. Finally, we compared the Casimir energy and Casimir force in the background before and after radiation respectively. We found that the Casimir force and Casimir energy got larger in the black hole background after tunneling, because of the self-gravitation. As far as the further physical meaning, it needs to be researched afterward.

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REFERENCES

Capri, A. Z., Kobayashi M., and Lamb, D. J (1996). *Classical and Quantum Gravity* **13**, 179.
Casimir, H. B. G. (1948). *Koninklijke Nederlandse Akademie Wetenschappen Proceedings* **51**, 793.
Chen, D. Y., Jiang, Q. Q., Li, H. L., and Yang, S. Z. (2006). *Chinese Physics* **15**(7), 1425.
Christodoulakis, T., Diamandis, G. A., and Getorgalas, B. C. (2001). *Physical Review D* **64**, 124022.
Elias, C. and Vagenas, E. C. (2003). *Physical Review D* **68**, 024015.
Gao, X. Q. and Zhu, J. Y., gr-qc/05011049.
Hang, H. R., Zhou, B. J., and Zhang, Z. J. (1994). *Quantum field in curved spacetime*, Huazhong Normal University Press, Wuhan.
Jiang, Q. Q., Yang, S. Z., and Li, H. L. (2006). *Communications Theory Physics* **45**, 457.
Setare, M. R. (2001). *Classical Quantum Gravity* **18**, 4823.
Wald, R. M. (1977). *Communications in Mathematical Physics* **54**, 1.
Wald, R. M. (1978). *Physical Review D* **17**, 1477.
Yang, S. Z., Jiang, Q. Q., and Li, H. L. (2005). *Chinese Physics* **14**(12), 2411.